Fostering Ethical Awareness in Differential Calculus with Optimization Problems

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1 License

2 Introduction

This activity is part of the NSF-funded project, *Normalizing Ethical Reasoning in Mathematics as a Foundation for Ethical STEM* (NSF Awards 220423, 2220395, 2220314). The goal of this initiative is to integrate ethical reasoning into mathematics instruction in a way that is accessible, meaningful, and relevant to students across STEM disciplines. The problems in this activity are designed for use in a differential calculus course, particularly in the context of optimization.

The ethical reasoning parts were designed to emphasize one of the ethical reasoning knowledge, skills, and abilities (KSAs), as described by author Rochelle E. Tractenberg in *Ethical Reasoning for a Data-Centered World*. In particular, the ethical reasoning prompts emphasize KSA 1, which is to "Identify and 'quantify' your prerequisite knowledge". This includes recognizing the assumptions, examination of benefits and harms to affected parties, and contextual understanding necessary to responsibly apply mathematical models.

The ethical reasoning components are intended to foster classroom discussion and reflection. When application problems include ethical reasoning, students are able to contextualize mathematics in the real world and can see how using math can affect them in their lives and future careers. Instructors may choose to introduce these problems during class as ungraded group activities, followed by whole-class discussion. Alternatively, the problems can be adapted for homework assignments or collaborative projects. This flexible structure allows instructors to scaffold ethical reasoning gradually, helping students build confidence in identifying stakeholders, evaluating assumptions, and considering the broader implications of mathematical decisions.

The problems are given in Section 3 below. To get solutions and commentary for these problems, contact the author by email to request them.

3 Exercises

1. The energy output of a solar panel is modeled by

$$E(\theta) = 100\cos(\theta - 30^\circ),$$

where θ is the angle of the panel in degrees.

   (a) Find the angle θ that maximizes energy output.

   (b) What is the maximum energy output?

   (c) What is a real-world situation where solar panels are used? List at least two parties affected by the panel placement.

   (d) Choose one stakeholder and describe a potential harm or benefit they might experience.

   (e) What professional or environmental guidelines might apply in this situation?

   (f) What data or technical knowledge would you need to validate the model used here (e.g., sun angle data, solar panel specifications, seasonal variation)?

2. A company wants to design a cylindrical container with a fixed volume of 1000 cm^3 that uses the least material.

   (a) Write a function for surface area in terms of radius r, and find the dimensions that minimize surface area.

   (b) What do your results mean in terms of material usage and cost?

   (c) What assumptions are you making about the material properties and environmental impact?

   (d) Assume that you are a packaging engineer for the company. Identify the prerequisite knowledge needed to responsibly use this model.

   (e) What sustainability guidelines or packaging regulations might apply to the choice of material used to mass produce this cylindrical container?

   (f) What assumptions are you making about the scalability of this design for mass production?

3. Coastal towns are vulnerable to tsunamis. In the event of a tsunami warning, residents must evacuate quickly to higher ground. One of the main evacuation routes is a highway that leads inland toward higher ground. Suppose a coastal city is considering improvements to the highway access points to maximize the number of vehicles that can evacuate per minute. However, increasing the number of vehicles on the road can

reduce the average speed due to congestion. Your task is to model this trade-off and recommend an optimal number of vehicles per minute that should be allowed onto the highway to maximize the evacuation flow. Let x represent the number of vehicles entering the highway per minute and let

$$F(x) = \frac{x(60 - 0.5x)}{60}$$

represent the number of vehicles that can travel one kilometer on the highway.

(a) Find the value of x that maximizes the flow rate. What is the maximum number of vehicles per minute that can travel 1 km on the highway?

(b) Would you recommend limiting the number of vehicles entering the highway to the optimal value from your model? Why or why not?

(c) Identify the prerequisite knowledge needed to responsibly use this model.

(d) List two affected parties and describe a potential harm or benefit they might experience based on your recommendation.

(e) What additional data would improve the accuracy of this model if it were included?

4. In a large city where the average monthly rent (2025 rent statistics) for a one-bedroom apartment is \$1,624, a local housing authority is considering implementing a rent control policy to cap rent increases. The goal is to ensure affordability while maintaining enough incentive for landlords to continue offering rental units. Suppose the number of available rental units N in the city is a function of the monthly rent r, modeled by

$$N(r) = -2(r - 1250)(r - 1150).$$

This function estimates the number of units available at different rent levels, assuming that too-low rents discourage landlords and too-high rents reduce affordability.

(a) Find the rent level r that maximizes the number of available rental units. What is the maximum number of units available?

(b) Describe some potential harms or benefits an affected party might experience based on decisions the housing authority makes.

(c) What housing policies, legal standards, or economic principles should be considered when interpreting the model and making a recommendation?

(d) What economic or demographic data would be necessary to responsibly use this model in policy-making?