Ethical Reasoning Lesson Plan

Title: Canned Food Optimization Project with Ethical Reasoning
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Course(s) and textbook(s) (or other info to contextualize the course and activity):
Calculus 1, Stewart Calculus
Type/Size of Institution(s): Designed for Community College (classes of 30), also intended for high school or University discussion section use.
Class Format: small groups of 2-3 or 3-4 with some whole-class discussion
AP Standards Addressed:
AP Topic: 5.11 Solving Optimization Problems
AP Skills: 1.D Identify an appropriate mathematical rule or procedure based on the relationship between concepts
3.F Explain the meaning of mathematical solutions in context
KSA's addressed: KSA2. Recognize an ethical issue KSA4. Identify & evaluate alternative actions, KSA5. Make & justify a decision, KSA6. Reflect on the decision
Learning Objective(s):
Students will be able to:

- Explain why optimization is useful in a real world situation
- Optimize a real world situation using multiple equations and derivatives
- Calculate the amount of carbon dioxide that can be produced/avoided for a real world scenario
- Consider the impacts of proposed changes to the manufacturing process on stakeholders

Time Required & Implementation Plan: Two hours
Grading and Assessment Recommendations: Final submission is graded using specifications-based grading, so the criteria for a successful submission is known to the students, and they receive full credit if all areas are addressed, and if anything is not addressed, they need to resubmit it.
Required resources and technology: Desmos or GeoGebra Graphing Calcualtor
Before class instructions: Please bring in some canned food in a steel can (not a beverage, which are in aluminum cans) to class for today's activity.
Brief Description/Abstract: In this activity, students conduct a stakeholder analysis to determine the impact on stakeholders of the carbon savings associated with creating a more optimal steel can.

Acknowledgments: Some source material for this project is inspired by Sam Shah's blog. Jamie Ryan also helped read some early drafts of the optimization part of the lesson. Furthermore, thank you to Catherine Buell, Victor Piercey, and Rochelle Tractenberg for feedback on an early draft of this lesson plan.

I. Lesson Launch

1. Show video: [Why steel is our most important (and dirtiest) metal] (13:18)
2. Have students make a list of key points from the video. Some example key points:

- The greenest way to make steel is through the use of hydrogen, however, it also requires renewable energy to power the hydrogen plants.
- In addition, hydrogen has to be transported from hydrogen manufacturing facilities
- One alternative is carbon capture, where carbon dioxide can be captured and reused or stored, to prevent release into the atmosphere.
- In addition to hydrogen and carbon capture, there is still a significant role in optimizing our usage of steel and using only what is needed to make the things we want to make.

II. Stakeholder Analysis

1. Questions for students to consider: In manufacturing cans, what kind of concerns might there be in minimizing costs? What kind of concerns are there for sustainability? In terms of workplace conditions and worker safety? How might these concerns overlap? How might these be different? Once you have listed the concerns, propose some solutions. Be sure to include optimizing our use of steel as one of the proposed solutions.

For each proposed solution, consider the impacted parties and the harms and benefits to each impacted party. Organize your thoughts on each proposed solution into a table such as:

Solution 1: Optimizing our use of steel in order to reduce steel consumption.

Impacted Party	Harms	Benefits

Solution 2: [Add your next proposed solution here]

Impacted Party	Harms	Benefits

How do the concerns of each party overlap and how are they different? What is the mathematician's role in helping to implement each of these solutions? As a mathematician, how do your ethical obligations overlap and differ from the concerns of the other parties?

2. Some example discussion points:
    - The goal of using as little source material as possible is common to both minimizing costs and environmental sustainability, as the raw materials have both a financial and environmental cost.
    - Canned food is sold primarily in steel cans, at least in the United States.
    - According to www.theworldcounts.com, production of steel is the most energy-consuming and CO_2 emitting industrial activity in the world.

- Producing steel emits carcinogenic air pollution and water pollution.
- However, other plans to reduce pollution might not necessarily result in cost savings, such as redesigning manufacturing processes to reduce pollution could have considerable upfront costs. (This is why things like carbon taxes and/or incentives for reducing pollution might be important. Also, the cost of fossil fuels is going up, as the video points out.)
- There might also be additional labor costs entailed in both reducing the source material consumed and in terms of creating processes that have less pollution.
- Methods of reducing pollution, such as using hydrogen, also have numerous safety concerns (but of course, doesn't working with carcinogenic pollutants too?) See https://www.elsevier.com/connect/addressing-the-process-safety-concerns-of-hydrogen-in-a-net-zero-economy
- Another possible solution to reducing pollution is recycling.This can include both recycling during the manufacturing process, such as when the circular lids are stamped out from sheets, as well as recycling used steel.
- However, although we can have these grand plans to create carbon-neutral steel production, another way that we can reduce these emissions is by finding more efficient ways to produce the cans themselves. This is where minimizing the surface area of the canned food containers comes in.

III. Minimizing the surface area of canned food:

(This part is adapted from https://samjshah.com/2012/05/31/a-calculus-optimization-poster-project/)

Materials needed: a small ruler for each group

This part of the lesson involves students working up at whiteboards (in groups of 2-3) or at desks or tables (groups of 3-4). Rather than giving students a worksheet, the intent is to explain to students the overall goal and give them space to play with the needed mathematics, prompting them as needed with the prompts in the first column. The second column shows the formulas students need to figure out. The third column shows follow-up questions to ask students as you circulate and speak with them. The fourth column lists common errors that students may make and how the teacher might respond to those errors.

Potential prompts to give to students	Formulas students will need to figure out	Follow-up questions	Common errors that may be made and how to respond
Objective: We are looking to find the minimal surface area for the volume of your particular can.			
This volume will remain constant. Find this volume.	V = pi*(r^2)*h	Why are we multiplying by h? (We are stacking layers of circles to find the volume) What are the units of the volume? (cubic units, either cubic centimeters or cubic inches.) Did you find the radius, or the diameter? What is the difference between the two?	Using the volume of the food, which is generally less than the volume of the can (Point out that you're looking for the volume of the can.) Students might use the wrong formula (Talk them through how to find the formula, have them look up the volume of a cylinder, or have them look at another group's work.)
Measure the height and radius of your can and find the current surface area of the can.	SA = 2*pi*r*h + 2*pi*r^2	Why do we multiply the area of the circle by 2? (We need to account for both the top and bottom of the can!)	Students may forget about the top and bottom of the can (have them look at another group's work or point out that they're missing a part of the can and see if they can find it)
Find an equation for the surface area of the can and the volume of the can.	V = pi*(r^2)*h SA = 2*pi*r*h + 2*pi*r^2	Which things vary? Which things remain the same? (Radius and height vary, surface area varies, volume remains constant.)	Students may be still using the r and height from their original can (help them to understand that we are looking for a new ideal r and h, along with surface area, to contain our known volume.)
Which one are we trying to minimize? (we call this the main	Trying to minimize surface area, so this is the main equation,		

equation and the other the helper equation)	and the volume is the helper equation.		
Substitute the helper equation into the main equation	Students will solve the helper equation for h: h = V/(pi*r^2) And then substitute it into the surface area equation: SA = 2*pi*r*(V/(pi*r^2)) + 2*pi*r^2	Why can we use a fixed value for V here? (Volume remains constant for our can throughout the whole exercise.) Why can't we substitute our known values of r and h? (Those values will change for the ideal surface area.)	Students may try to solve the helper equation for r, which works but is more complicated. (Ask them if there would be an easier variable to solve for in the helper equation, especially if they are getting stuck)
Find the minimum of the main equation (treating it as a function)	Students will take derivative of SA equation and set equal to 0 to find relative minimum.	Are you sure this is a relative minimum and not a relative maximum? Why or why not? Is the relative minimum you found the same as the absolute minimum? (Will need to graph to verify.)	
Use percent error formula for figuring out how different the ideal surface area is from the original surface area.	Percent = (Original SA – Optimal SA)/Optimal SA * 100		
For your ideal can, what is the height? What is the radius? What is the ratio of height to radius?	We have found a value for r, so using the known V, we can then calculate h. Then calculate h/r, the height to radius ratio.		

Follow-up discussion:

- What might be some practical issues involved in creating a more efficient can?
- Why might it not be possible to implement the most optimal can?
- What are the impacts of various possible changes on stakeholders?

Some things to consider (source: Stewart Calculus):

- If the cylinders come from square pieces of metal, but the lids come from squares of side length 2r, then h/r is approximately 2.55.
- How could we fit more lids onto the same square piece of metal? (One strategy is hexagons, which gives h/r of 2.21).
- Why would different products be in different shaped cans? What are some other factors that can producers might keep in mind? (Why are tuna cans so short? Why is evaporated milk different from different things?) Portion sizes, for example? Shipping?

IV. Carbon Footprint Activity Worksheet:

In this activity, you will find the carbon footprint that is saved by the optimization you did in the last activity

1. According to https://www.sustainable-ships.org/stories/2022/carbon-footprint-steel,

There are two main methods of processing steel:

- Blast Furnace-Basic Oxygen Furnace (BF-BOF) referred to the primary path, which uses 13.8% scrap steel and has emissions of 1.987 tons of CO_2 / ton of steel
- Electric Arc Furnace (EAF), or the secondary path, which uses 105% scrap steel and has emissions of 0.357 tons CO_2/ ton of steel. (It is greater than 100% since some of the scrap steel is used up during the secondary path.)

2. Information about weights of sheets of steel can be found here: https://en.metcalc.info/calc-rolled/sheet/

Record the appropriate weight in this space.

3. Next, consider transportation: Carbon footprint for transporting goods via bulk is approximately 7.9 grams per tonne-km. This means that shipping 1000 tons of steel halfway across the world (10.000 km) would result in a carbon footprint of 79 metric tons of Co2.

4. Assuming we were using the primary path, and it was indeed possible to make the optimized can you came up with, what would be the carbon footprint benefits for your optimal can? You'll need to estimate how many cans of your particular food are sold per year.

The food that I am using:
The number of cans sold per year:
The number of sheets of steel needed:
The number of tons of steel per year:

Which did you choose: BF-BOF or EAF?
How many tons of CO_2 would be needed to make your steel?

How many tons of Co2 would be needed to ship it?

5. In one year a mature tree will absorb more than 48 pounds of carbon dioxide from the atmosphere and release oxygen in exchange. https://www.usda.gov/media/blog/2015/03/17/power-one-tree-very-air-we-breathe.

Find how many trees would you need to plant to offset the carbon if you didn't make the modifications to your can:

V. Final submission handout

As a group, write one paragraph about what you learned from this project. Who are the stakeholders in the issue, and how does the Calculus tool of optimization help to address each of the stakeholder concerns? What are the steps for carrying out an optimization, and what is the role of the helper equation and the relative minimum in this process? Is it possible to make the optimal can, and what are some practical constraints in doing so? Furthermore, if it is possible to shift to carbon neutral processes such as Hydrogen, is there still a role for reducing the amount of steel needed to make a can? If so, what are the impacts of doing so on each of the stakeholders? How does a mathematicians ethical obligations differ from the concerns of other stakeholders?

Submit this paragraph along with a clean copy of your work from the problem (as in, choose one can and show the fully worked out problem for that can, including all steps and equations, and your final percentage and height to radius ratio.

To receive credit for this final submission, make sure you address all items on this checklist.

Specifically, ensure your paragraph:

- ☐ Lists stakeholders from earlier in the activity and addresses how to use optimization to help each stakeholder
- ☐ Summarizes each step in the optimization process, including the role of the helper equation and relative minimum
- ☐ Explains the practical constraints in making the optimal can
- ☐ Addresses the role for reducing the amount of steel even if we switch to Hydrogen
- ☐ Addresses the reduction of the amount of steel on each stakeholders